WEAK NONLINEARITY FOR STRONG NOMNORMALITY

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We propose a theoretical approach to derive amplitude equations governing the weakly nonlinear evolution of nonnormal dynamical systems when they experience transient growth or respond to harmonic forcing. This approach reconciles the nonmodal nature of these growth mechanisms and the need for a center manifold to project the leading-order dynamics. Under the hypothesis of strong nonnormality, we take advantage of the fact that small operator perturbations suffice to make the inverse resolvent and the inverse propagator singular, which we encompass in a multiple-scale asymptotic expansion. The methodology is outlined for a generic nonlinear dynamical system, and several application cases which highlight common nonnormal mechanisms in hydrodynamics: the streamwise convective nonnormal amplification in the flow past a backward-facing step, and the Orr and lift-up mechanisms in the plane Poiseuille flow. For the two-dimensional flow down a backward-facing step of expansion ratio 2 and Reynolds number Re=500 (Fig. 1a), the linear response $u_o(x,y)$ to harmonic forcing (Fig. 1c) is maximum at a nondimensional frequency of 0.08 (Fig. 1d) for an optimal forcing structure $f_o(x,y)$ given in Fig. 1b, with a gain of approximately $G_{lin}(\omega = 2\pi 0.08) \approx 7.10^3$, as computed in the literature [1, 2]. In order to investigate the effect of nonlinearities under a small forcing $F\epsilon^3 f_o$, we use the following Ansatz for the response $u = \epsilon A(t/\epsilon^2)u_0(x,y)\exp(i\omega t)$, where $\epsilon = G_{lin}^{-2}$ is the small parameter and demonstrate that the amplitude A is governed by the following forced Landau equation

$$\epsilon^2 \frac{dA}{dt} = \gamma F - \eta A - \mu |A|^2 A, \tag{1}$$

where the parameters γ, η and μ are given as closed form expressions involving only the solutions of linear systems. The resulting weakly nonlinear predictions regarding the response saturation in amplitude match direct numerical simulations well at different forcing frequencies (Fig. 1d).

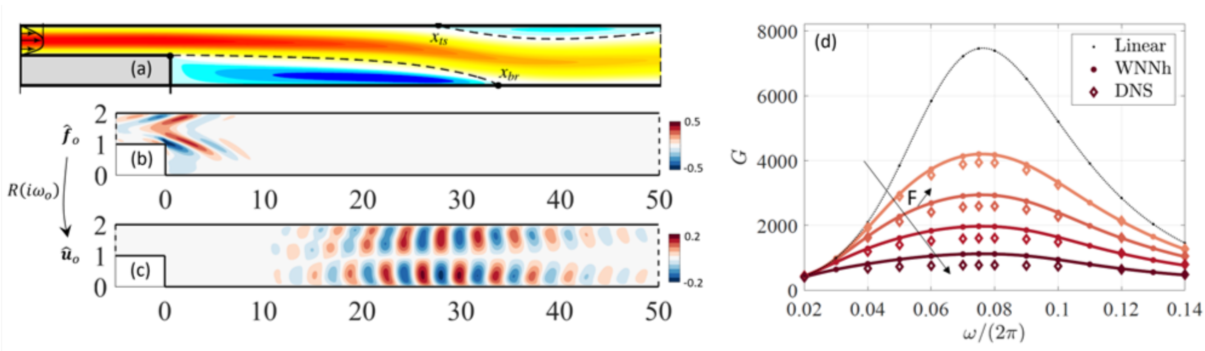

Figure 1: (a) Backward facing step flow at Re=500, (b) optimal forcing at $\omega = 2\pi 0.08$, (c) associated permanent response and (d) gain for different forcing intensities F as obtained from Eq. 1 (WWNh) and from DNS.

References

[1] E. Boujo and F. Gallaire. Sensitivity and open-loop control of stochastic response in a noise amplifier flow: the backward-facing step. *J. Fluid Mech.*, **762**, 2015.

[2] V. Mantic-Lugo and F. Gallaire. Self-consistent model for the saturation mechanism of the response to harmonic forcing in the backward-facing step flow. *J. Fluid Mech.*, **793**, 2016.